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ANALOGY BETWEEN A THERMISTOR AND A PLASMA GENERATOR ARC

A. S. Sergienko

UDC 533.9.082.15

An analysis of the experimental data confirms the existence of an analogy between a semiconductor thermistor and a dc electric arc burning under linear plasma generator conditions.

In order to develop engineering methods of designing electrical circuits containing nonlinear elements it is important to draw an analogy between metallic and semiconducting thermistors and a "gaseous" temperature-dependent resistance — a dc electric arc. A preliminary qualitative analysis of the experimental data shows that, despite the difference in physical properties and internal processes, there are a number of common features, based on the sensitivity of the resistance to temperature changes.

There follows a qualitative and quantitative evaluation of the correspondence between the two elements. For comparison, all the integral thermal and electrical characteristics of the thermistor and the arc are treated in accordance with the procedures developed in the theory of solid temperature-dependent resistances. Parallels are drawn in the following order: static volt—ampere characteristics — region of stable operation — circuit power supply regime — sensitivity of resistance to changes in gas flow rate — temperature — power output.

For purposes of comparison we will use an electric arc vortex-stabilized in a cylindrical channel. The arc chamber [1], open at one end (Fig. 1), is formed by two tubular coaxial electrodes ($d_a = 8 \cdot 10^{-3}$ m, $l_c = 0.1$ m, $d_c = 5 \cdot 10^{-3}$ m, and $l_a = 0.1$ m), electrically insulated from each other by a ventilated air gap ($\delta = 1.5 \cdot 10^{-3}$ m). The gas is supplied tangentially to the working space through two diametrically opposed orifices ($d = 1 \cdot 10^{-3}$ m).

The thermistor and the electric arc are both characterized by a relationship between thermal and electrical phenomena expressed by the volt—ampere characteristic. Both for a type KMT-1 thermistor ($R_{20} = 102.8$ kΩ, $B = 4225$°K, and $T_m = 20$-80°C) in an air medium ($w = 0$) [2] and an electric arc [1] the dependence of the current on the voltage drop is nonlinear and the differential resistance is negative $(dU/dI) < 0$.

Graphical differentiation of the volt—ampere characteristics established that

$$\left(\frac{dU}{dI}\right)_{\text{arc}} = -(0.25-0.67) \leqslant \left(\frac{dU}{dI}\right)_{\text{t}} = -(1-3000).$$

Clearly, the slope of the volt—ampere characteristic for the thermistor may vary more broadly than for the electric arc. The specific properties and characteristics of the resistors compared become especially apparent when the region of stable operating regimes is considered. Under steady-state conditions, the heat transfer between the thermistor and an ambient medium of normal density obeys the law of convective energy transport

$$I_{\text{t}}^2 R_{\text{t}} = K(T - T_{\text{m}}). \tag{1}$$

A. V. Lykov Institute of Heat and Mass Transfer, Academy of Sciences of the Belorussian SSR, Minsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 32, No. 5, pp. 878-885, May, 1977. Original article submitted December 15, 1975.

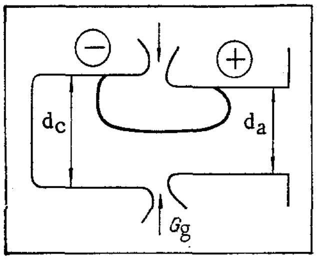

Fig. 1. Diagram of electric arc chamber. $d_c = 8 \cdot 10^{-3}$ m, $d_a = 5 \cdot 10^{-3}$ m.

When the heat balance (1) is disturbed as a result of a change in the current in the circuit $[I^2R_t < K(T - T_m)]$, as the temperature of the thermistor decreases its resistance R_t increases. On the descending branch of the volt—ampere characteristic the limit $R_t = R_{t,max}$ ($U = U_{max}$, $I = I_{min}$) corresponds to the extremum ($dU/dI = 0$). The line joining the maxima of the volt—ampere characteristics and the coordinate origin bounds the region of static stability of the thermistor on the left (U_{max} boundary).

When $I^2R_t > K(T - T_m)$ the resistance of the thermistor R_t falls as a result of the increase in temperature. The minimum $R_t = R_{t,min}$ corresponds to the maximum permissible thermistor temperature $T = T_p$. For example, for a type MT-57 thermistor $T_p = 150$°C [2]. The straight line corresponding to the limiting temperature at $U = U_{min}$, $I = I_{max}$, passing through the origin, bounds the region of equilibrium states of the thermistor from below (I_{max} boundary). The upper boundary is the volt—ampere characteristic ($w = w_{max}$ = const), whose position is determined by the rated voltage of the power source ($U_t = U_{arc} - I \cdot R_b$).

The above considerations are essentially qualitative, since no experimental data on the determination of the region of stable operation of a thermistor could be found in the specialized literature. As regards the electric arc, the theory has not yet reached the stage at which it is possible to calculate the mechanism of a heavy-current arc discharge under plasma generator conditions. Accordingly, the limiting arc burning regimes will be considered on the basis of the experimental data [1, 3].

In a plasma generator, under the influence of the gasdynamic forces the positive column of an arc with self-adjusting length is continuously deformed in the longitudinal and transverse directions. Part of the responsibility for this can be attributed to the arc spots which are in complex motion over the surface of the electrode. Under the influence of the opposing forces of the air flow and periodic breakdown of the arc—electrode or arc—arc type the spots move along a circular trajectory and scan in the longitudinal direction.

As the current falls to $I = I_{min}$ at constant gas flow rate G_g the arc is stretched until it collapses. The values of the current $I = I_{min}$ and voltage $U = U_{max}$ recorded at the moment of collapse define the upper boundary of the region of stability (U_{max} boundary). As the current increases to $I = I_{max}$ (G_g = const) the arc is pulled into the air gap between the electrodes and seriously erodes them. These conditions ($U = U_{min}$, $I = I_{max}$) correspond to the lower boundary of the region of stable arc-burning regimes. The current limits at $G_g > 0.35 \cdot 10^{-3}$ kg/sec are expressed by the line $I = I_p = 200$ A, which closes the region of stability on the right.

Thus, with respect to the limiting values of the voltage and current the thermistor and the arc are qualitatively identical.

Stability of the current in the circuit in the presence of a nonlinear element with negative differential resistance ($dU/dI < 0$) can be achieved either by means of a steeply falling external power source characteristic or by means of an auxiliary resistance R_b. On the one hand, R_b is necessary for the sake of stability, while on the other it is undesirable as an element that nonproductively dissipates the energy of the power source. Accordingly, it is important to establish the optimal relationship between the resistance of the nonlinear element and the auxiliary resistance.

Within the region of stable operation ABC at constant gas flow rate G_g = const the relation $R_{arc} = f(R_b)$ (Fig. 2) is nonlinear and has a positive slope ($dR_{arc}/dR_b > 0$). In order to calculate the arc resistance in the static regime the following expression, obtained for plasma generators of similar design, has been proposed [3]:

$$R_{arc} = A^a U^{-b} I^{-b} G_g^b d_a = A^a N_{arc}^{-b} G_g^b d_a, \tag{2}$$

where $a = 3.663$, $b = 2.663$, and $A = 6.46 \cdot 10^4$.

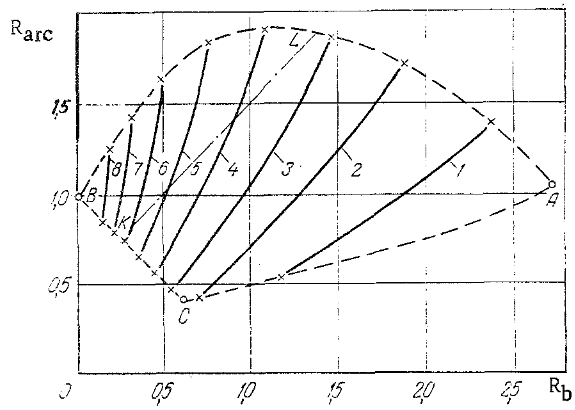

Fig. 2. Static resistance of the electric arc as a function of the resistance of the ballast rheostat. ABC denotes the region of stable arc-burning: 1) $G_g = 0.25 \cdot 10^{-3}$ kg/sec; 2) 0.35; 3) 0.5; 4) 0.6; 5) 0.75; 6) 0.85; 7) 1.0; 8) 1.1. KL represents the line $R_{arc} = R_b$. R_{arc}, R_b in Ω.

As the gas flow rate increases from $G_{g_1} = 0.25 \cdot 10^{-3}$ kg/sec to $G_{g_2} = 1.1 \cdot 10^{-3}$ kg/sec, the slope increases $(dR_{arc}/dR_b)_{G_{g_1}} < (dR_{arc}/dR_b)_{G_{g_2}}$, i.e., the arc resistance R_{arc} depends increasingly on R_b. The upper boundary curve AB has a singular point $(dR_{arc}/dR_b) = 0$ with coordinates $R_{arc} = 1.92$ Ω, $R_b = 1.58$ Ω at $G_g = 0.6 \cdot 10^{-3}$ kg/sec. The straight line KL ($R_{arc} = R_b$) divides the region ABC into two zones: BKL ($R_{arc} > R_b$) and KLAC ($R_{arc} < R_b$). If we introduce the dimensionless coefficient

$$\gamma = \frac{R - R_b}{R + R_b} = \frac{U - U_b}{U_{arc}}, \tag{3}$$

characterizing the circuit power supply regime [4], then in the zone BKL $\gamma_{arc} > 0$, and in the zone KLAC $\gamma_{arc} < 0$. In the general case the coefficient γ may vary from +1 ($R_b = 0$) to −1 ($R_b \gg R$). As a result of calculations based on (3) we obtained the following quantitative relations: for the arc $\gamma_{arc} = +0.3$ to −0.513; for the thermistor $\gamma_g = +0.97$ to −0.97 [4]. Thus the γ for the arc and the thermistor are of roughly the same order ($\gamma_{arc} \approx \gamma_t$).

The resistance of the nonlinear temperature-dependent element depends to a considerable extent on the heat-transfer conditions. We will employ the concept of a convection sensitivity factor

$$\nu = \frac{1}{R} \cdot \frac{dR}{dw} 100\%, \tag{4}$$

which characterizes the change in relative resistance due to a change of 1.0 m/sec in the gas flow velocity. For the arc, in accordance with (4), the coefficient ν_{arc} takes values (0.12-0.21)% on the range of gas flow rates $G_g = (0.35\text{-}1.0) \cdot 10^{-3}$ kg/sec, which corresponds to a mean mass flow rate $w_{mm} = G_g/\rho F = (280\text{-}800)$ m/sec. Similar calculations for the thermistor [2] lead to the value $\nu_t = (0.576\text{-}8.14)\%$ at $w = (1\text{-}20)$ m/sec. Hence it follows that, despite the lower flow rate ($w_{arc} \gg w_t$), the convection sensitivity factor for the thermistor is much greater than that for the arc ($\nu_{arc} \ll \nu_t$).

In the last analysis, all the parameters such as gas velocity, current, pressure, etc., affect the temperature of the nonlinear element and hence its resistance. Accordingly, the temperature characteristic R = f(T) (Fig. 3) is of central importance to the investigation.

The temperature characteristic of a metallic thermistor (straight line 1) is described by the following equation, common to this class of resistors [4]:

$$R_l = R_i [1 + \alpha (T - T_i)]. \tag{5}$$

For a semiconducting thermistor (curve 2) it has been shown [2, 4] that the resistance depends exponentially on temperature:

$$R_t = R_i \exp(B \cdot T^{-1}). \tag{6}$$

For both linear and nonlinear thermistors it is assumed that the temperature is uniformly distributed over the entire volume of the resistor.

In the electric arc a distinction is made between the temperature T_{ax}, measured on the axis of the positive arc column at a certain distance from the cathode or anode, and the mean

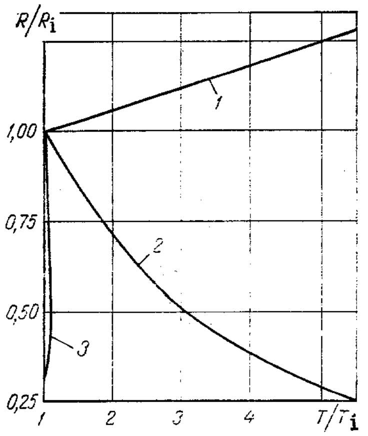

Fig. 3. Comparison of the relative temperature characteristics of various elements. 1) Linear element (R_i = 500 Ω, T_i = 20°C); 2) semiconducting thermistor (R_i = 1350 Ω, T_i = 20°C); 3) electric arc (R_i = 1.82 Ω, T_i = 4250°K, G_g = $0.75 \cdot 10^{-3}$ kg/sec).

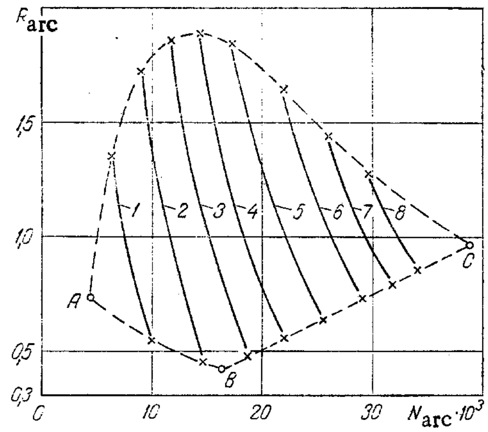

Fig. 4. Power dependence of the static resistance of an electric arc. ABC denotes the region of stable arc-burning: 1) G_g = $0.25 \cdot 10^{-3}$ kg/sec; 2) 0.35; 3) 0.5; 4) 0.6; 5) 0.75; 6) 0.85; 7) 1.0; 8) 1.1. R_{arc}, Ω; $N_{arc} \cdot 10^3$, W.

mass temperature $T_{mm} = f[H = (N_{arc} - Q)/G_g]$. In a conducting channel the arc temperature is a function of length and radius. For example, the temperature gradient dT/dr in the positive column, calculated by the method of stepwise approximation of σ(S) [5], for an argon arc [6] in a cylindrical channel with $d_m = 6 \cdot 10^{-3}$ m at I = (30–110) A and E = (636–860) V/m...* In view of the fact that no local temperature measurements were made, as the parameter we used the mean mass temperature T_{mm}. For the electric arc [1, 3] the temperature dependence of the resistance is given by the expression

$$R_{arc} = 0.86 \pm \left(0.71 - 4.71 \ln \frac{T_{mm}}{4350}\right)^{0.5}. \tag{7}$$

Correct to 3%, expression (7) describes the dependence $R_{arc} = f(T_{mm})$ (curve 3). As base values we took T_{mm} = 4250°K, R_i = 1.82 Ω (I = 96 A, U = 178 V, G_g = $0.75 \cdot 10^{-3}$ kg/sec, and N_{arc} = 17.1 kW). A feature of the $R_{arc} = f(T_{mm})$ graph is the presence of an extremum ($dR_{arc}/dT_{mm} = 0$) at T_{mm} = 5120°K. This behavior of the temperature characteristic of the electric arc is determined by the heat-transfer conditions in the discharge chamber. As the current increases from 96 to 160 A, the rate of total heat loss to the electrodes lags behind the rate of energy release in the arc column. With increase in the current to I = 200 A, as the losses to the electrodes lead the energy release in the arc, T_{mm} falls to T_{mm} = 4920°K, and the resistance R_{arc} drops to 0.65 Ω. An analysis of the curves in Fig. 3 reveals that as the temperature changes from 20 to 100°C the resistance of the semiconducting thermistor falls by a factor of 3.5 and that of the linear element, by a factor of 1.8. For the arc, a change in mean mass temperature T_{mm} on the interval (4250–5125)°K corresponds to a fall in resistance by a factor of 2.1.

In addition to the convection sensitivity factor, we introduce the temperature coefficient

$$\beta = \pm \frac{1}{R} \cdot \frac{dR}{dT} 100\%. \tag{8}$$

For a semiconducting thermistor (R_{20} = 1360 Ω), as distinct from a linear element, the resistance temperature coefficient has a variable and negative value β_t = −(1.1–1.64)% [4], while for the arc β_{arc} is variable both in magnitude and sign:

*This sentence appears to be incomplete in the Russian original — Translator.

TABLE 1. Comparative Characteristics of Thermistor and Electric Arc

N	Criterion	Electric arc	Thermistor	Results of comparison
1	Volt—ampere characteristic	$(dU_{arc}/dI)_{arc} < 0$	$(dU_t/dI)_t < 0$	$(dU_{arc}/dI)_{arc} \leq (dU_t/dI)_t$
2	Circuit equation	$U_{ci} = U_{arc} + I \cdot R_b$	$U_{ci} = U_t + I \cdot R_b$	Kirchhoff equation
3	Stability criterion	$R_{arc}^* + R_b > 0$	$R_t^* + R_b > 0$	Kaufmann criterion
4	Region of stability upper boundary lower boundary	 $U_{arc,max}$; $I_{arc,min}$ $U_{arc,min}$; $I_{arc,max}$	 $U_{t,max}$; $I_{t,min}$ $U_{t,min}$; $I_{t,max}$	 U_{max} boundary I_{max} boundary
5	Circuit power supply factor	$\gamma_{arc} \gtreqless 0$	$\gamma_t \gtreqless 0$	$\gamma_{arc} \approx \gamma_t$
6	Convection factor	$\nu_{arc} > 0$	$\nu_t > 0$	$\nu_{arc} < \nu_t$
7	Temperature factor	$\beta_{arc} \gtreqless 0$	$\beta_t < 0$	$\lvert\beta_{arc}\rvert < \lvert\beta_t\rvert$
8	Power factor	$m_{arc} < 0$	$m_t < 0$	$m_{arc} \ll m_t$

$$\beta_{arc} = (0.54 - 2.0)\cdot 10^{-1}\% \quad \text{for} \quad \frac{dR_{arc}}{dT_{mm}} < 0,$$

$$\beta_{arc} = -(1.0 - 1.2)\cdot 10^{-1}\% \quad \text{for} \quad \frac{dR_{arc}}{dT_{mm}} > 0.$$

Comparing these results, we conclude that, under the conditions specified, the temperature coefficients of the semiconducting thermistor and the electric arc differ considerably in magnitude ($|\beta|_{arc} < |\beta|_t$).

Another important quantity, in addition to the temperature parameter, is the power sensitivity factor, which indicates the sensitivity of the resistance to a 1-W change in power, in percent:

$$m = -\frac{1}{R} \cdot \frac{dR}{dN} \, 100\% . \tag{9}$$

At a constant gas flow rate within the region of stable operation ABC the $R_{arc} = f(N_{arc})$ dependence is a curve with a negative slope ($dR_{arc}/dN_{arc} < 0$) (Fig. 4). A change of arc power on the interval N_{arc} = (17-23) kW at $G_g = 0.75 \cdot 10^{-3}$ kg/sec causes the resistance R_{arc} to fall by a factor of 2.8. The power sensitivity factor for the arc, calculated from (9), is equal to m_{arc} = (0.026-0.0064)% at I = (70-200) A, U_{arc} = (73-195) V, $G_g = (0.25\text{-}1.1) \cdot 10^{-3}$ kg/sec, and for a type KMT-1 thermistor (R_{20} = 102.8 kΩ, B = 4225°K) [2] m_t = (0.77-1.02)% at 0.1 W < N_t < 0.3 W. Hence it follows that the power sensitivity factor for the arc is much less than for the semiconducting thermistor ($m_{arc} \ll m_t$).

The principal results of the analysis made to establish an analogy between the semiconducting thermistor and the dc electric arc are summarized in the Table 1.

Thus, the dc electric arc in a longitudinal air flow is a temperature-dependent resistor comparable to the semiconducting thermistor in respect of items 2-5 of the table, while differing significantly from the latter in respect of items 1 and 6-8.

NOTATION

U, voltage drop across resistor, V; I, resistor current, A; R, static resistance of resistor, Ω; d, arc electrode diameter, m; l, arc electrode length, m; r, arc channel radius, m; δ, interelectrode gap, m; G, gas flow rate, kg/sec; w, gas velocity, m/sec; T, temperature, °K; K, thermistor dissipation factor, W/deg; γ, circuit power supply factor; ν, convection sensitivity factor; B, constant depending on the properties of the thermistor material, °K; ρ, gas density, kg/m^3; F, cross section of internal channel of arc anode, m^2; H, gas enthalpy, J/kg; Q, heat losses to arc electrodes, W; α, temperature coefficient of resistance of linear resistor; β, temperature coefficient; m, power sensitivity factor; σ, electrical conductivity, $\Omega^{-1}\cdot m^{-1}$; S, thermal potential, W/m. Indices: arc, arc; c, cathode; m, medium; t, thermistor; g, gas; ax, axial; a, anode; mm, mean mass; i, initial.

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SOME STEADY-STATE PROBLEMS IN THERMAL CONDUCTIVITY FOR A THIN METAL WIRE HEATED BY AN ELECTRICAL CURRENT

A. N. Shcherban' and V. N. Tarasevich

UDC 621.365:621.372.061.6

Analysis is made of steady-state problems in thermal conductivity for a thin metal wire heated by an electrical current, including consideration of the temperature dependence of the thermophysical properties of the wire material and of the conditions for heat transfer with the surrounding medium.

The thermal conductivity of thin metal wires heated by an electrical current, which are widely used in the field of thermal measurement in the form of thermal sensors such as thermoanemometers, has been investigated in detail within the limitations of the linear problem where the thermophysical properties of the wire material (resistance, heat capacity, coefficient of thermal conductivity) and the conditions for heat transfer with the surrounding medium (coefficient of heat transfer) do not depend on temperature [1, 2]. Such a formulation of the problem has important theoretical significance; however, its solution is only valid for small excess temperatures and therefore can be used in practice for an extremely limited set of engineering problems.

In this paper, an analysis is made of the steady-state problems [1, 2] with consideration of a linear dependence on temperature for the resistance and for the coefficients of heat transfer and thermal conductivity of the wire.

If the heat transfer between the wire and the surrounding medium obeys Newton's law, the differential equation for steady-state thermal conductivity can be written in the form

$$\frac{d}{dx}\left[\lambda(t)\,\omega\frac{dt}{dx}\right] = -\frac{1}{\omega}I^2\rho(t) + \pi d\alpha(t)(t - t_b). \qquad (1)$$

In first approximation, let

$$\rho(t) = \rho_0(1 + \beta t);\ \ \lambda(t) = \lambda_0(1 + \delta t);\ \ \alpha(t) = \alpha_0(1 + \gamma t).$$

We then obtain from Eq. (1)

Institute of Technical Thermophysics, Academy of Sciences of the Ukrainian SSR, Kiev. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 32, No. 5, pp. 886-894, May, 1977. Original article submitted June 2, 1976.